MCTR903, Winter Term 12-13

Lecture 7 – Monday, November 24, 2012

Dynamic Compensation-II

Some slides of this lecture are based on materials from W. Bolton. *Mechatronics: A Multidisciplinary Approach*. 4th Edition, Pearson, 2008.

Objectives

When you have finished this lecture you should be able to:

- Understand the digital realization of the control modes.
- Understand different tuning methods of PID controllers.

Outline

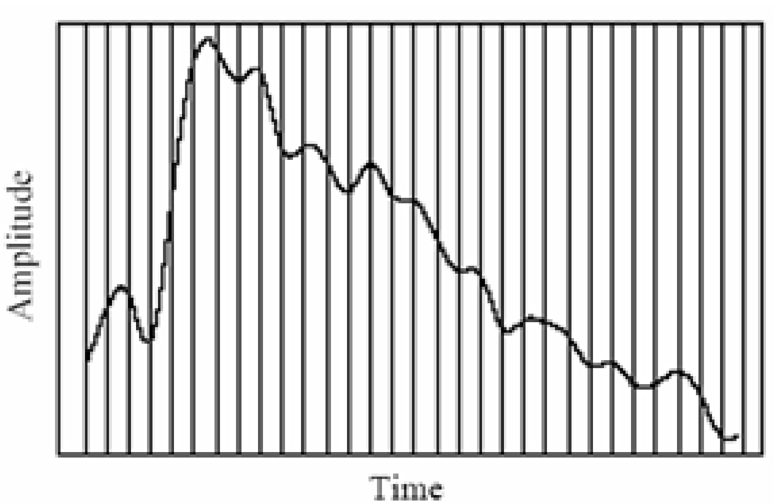
- Introduction to Digitization
- Digital Controllers
- Implementing Control Modes
- PID Tuning
- Summary

Outline

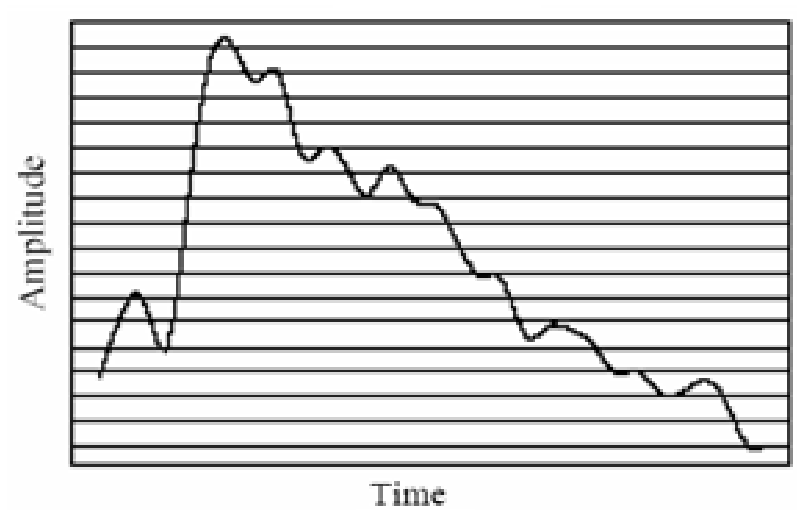
- **Introduction to Digitization**
- Digital Controllers
- Implementing Control Modes
- PID Tuning
- Summary

Introduction to Digitization

- **Digitization:** means conversion to a stream of numbers, and preferably these numbers should be integers for efficiency.
- To digitize, the signal must be sampled in each dimension: in time, and in amplitude.



Sampling the analog signal in the time dimension



Quantization is sampling the analog signal in the amplitude dimension

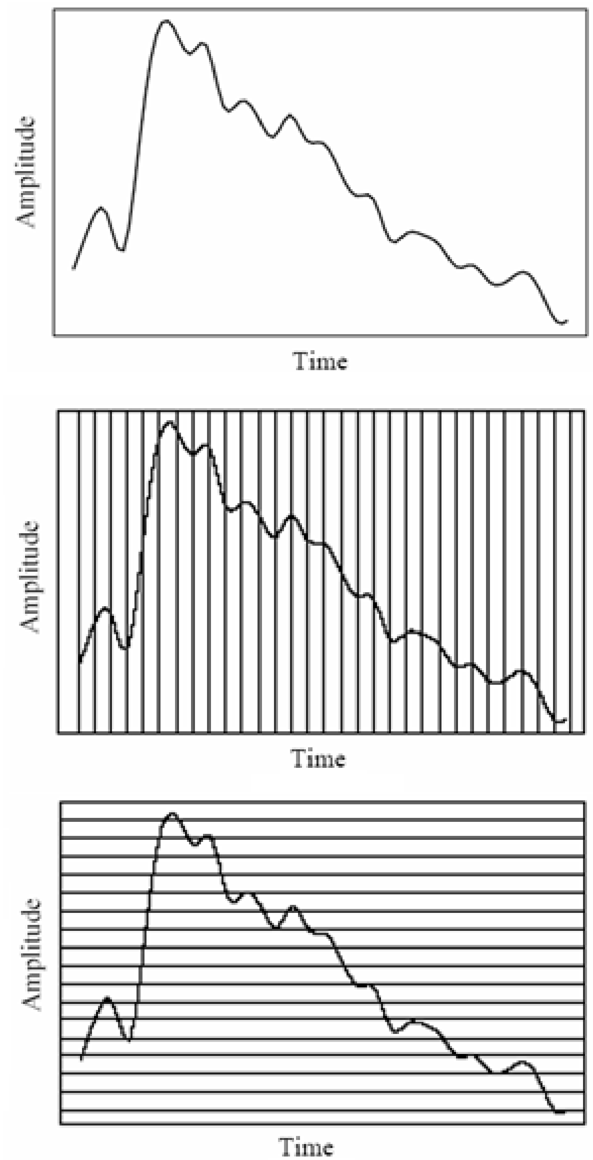
Introduction to Digitization

- **Digitization:**

  To decide how to digitize an analogue error signal for example, we need to answer the following questions:

  Q1: What is the sampling rate?

  Q2: How finely is the data to be quantized, and is quantization uniform?



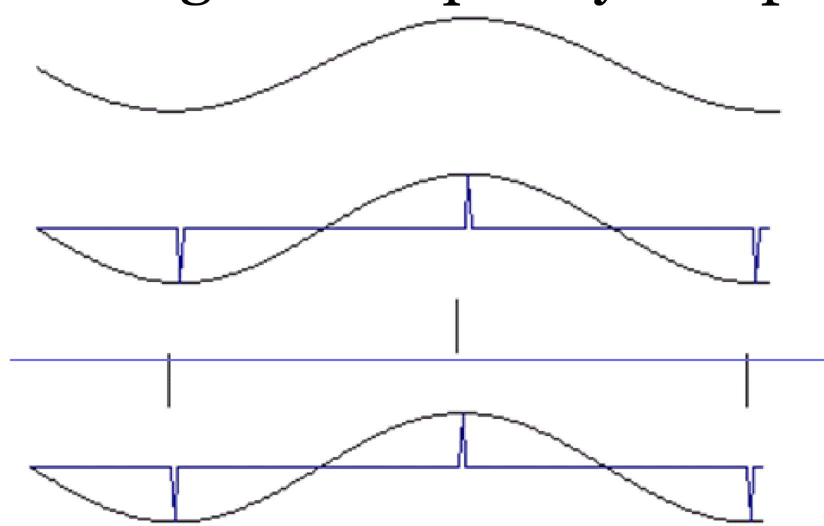
Introduction to Digitization

- **Sampling:**
  - ◇ **Sampling Theorem:**
    The minimum sampling rate for "correct" sampling depends on the frequency characteristics of the signal.
  - ◇ **Nyquist-Shannon Sampling Theory:**
    To distinguish unambiguously between all signal frequency components, sampling rate must be at least **twice** as high as the highest frequency component.

Harry Nyquist
(1889-1976)

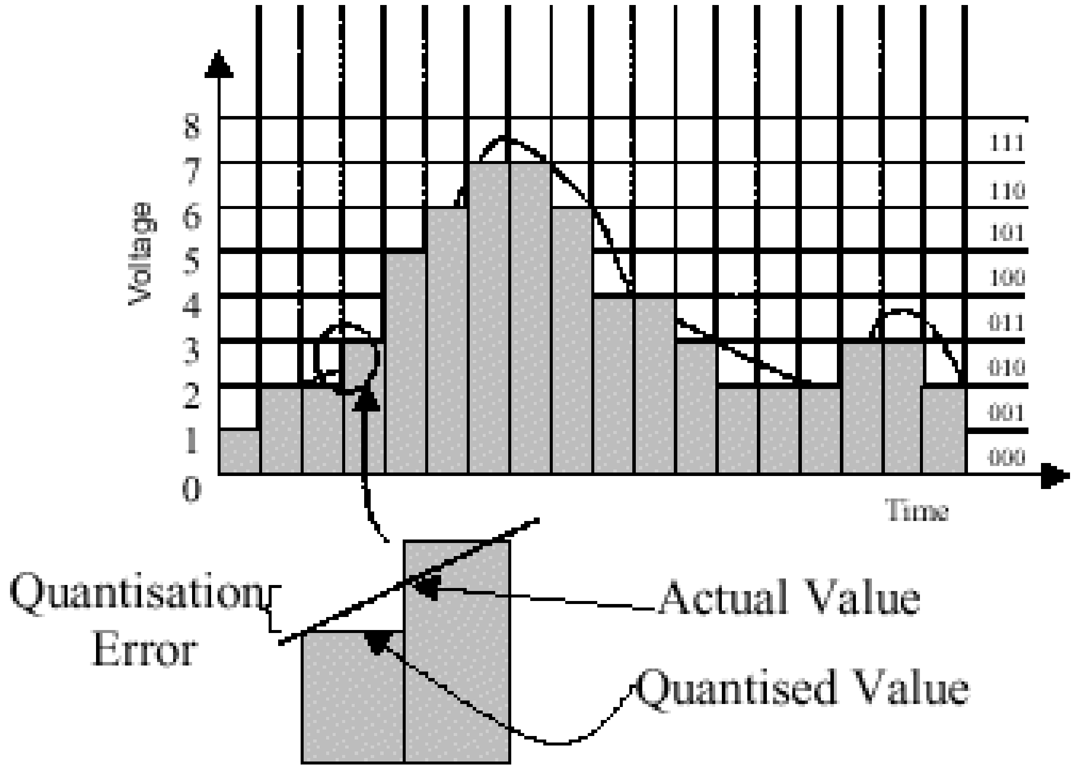
Claude Shannon
(1916-2001)

In digital control systems, the sampling rate is generally much higher than this (>20 times).



Introduction to Digitization

- **Quantization:**



\# of voltage levels=2^N

N is the resolution in bits.

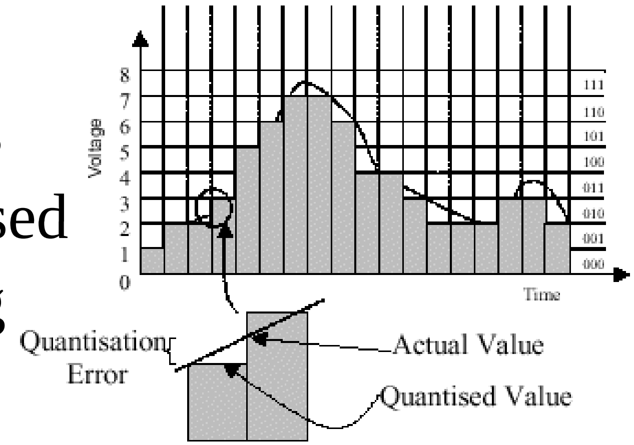
N=3 bits

◇ The **higher** the bits, the **closer** to the original waveform the signal become.

◇ But the **higher the bits**, the **larger the bandwidth**.

Introduction to Digitization

- **Quantization:**



Bitrate (sometimes written bit rate, data rate) is the number of bits that are conveyed or processed per unit of time. The bit rate is quantified using the 'bits per second' (bit/s or bps) unit.

Bitrate=N/T_s=$N F_s$

where T_s=Sampling Period; F_s=Sampling Frequency and

N= #bits to represent one quantized sample

Example: What is the bitrate of digitized error signal if quantized into 8 voltage levels with sampling frequency of 100Hz?

Solution: bitrate=NF_s=N/T_s,
#of levels =2^N or 8 =2^N so N=3 and bitrate=3x100=300 bps.

Introduction to Digitization

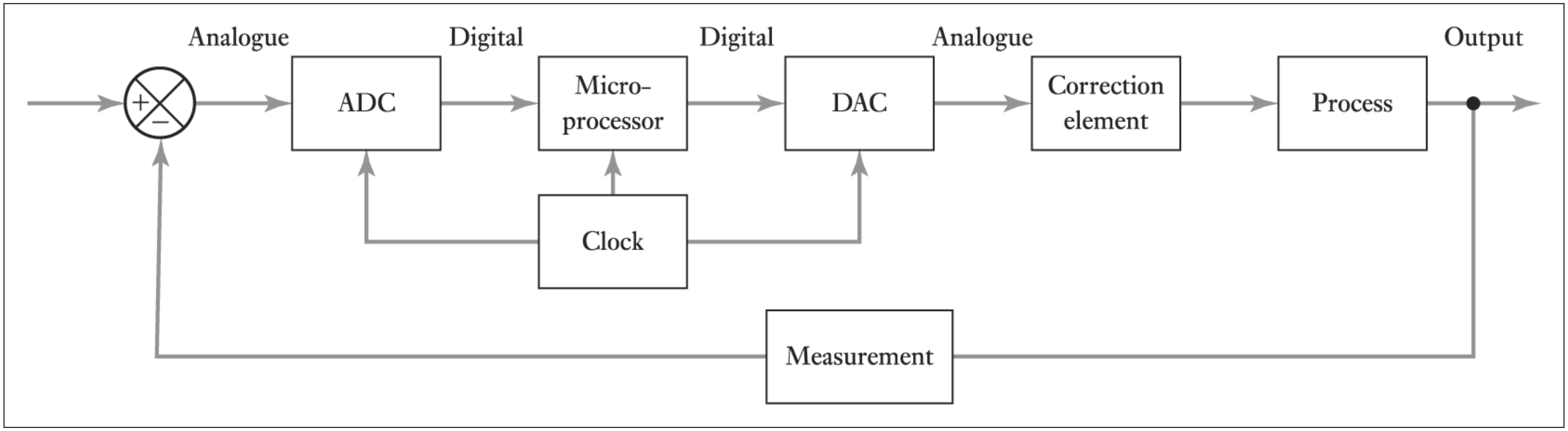
- **Digital Signals vs. Analogue Signals**

Signal	Advantages	Disadvantages
Digital	◊ Only digitized information can be transported through a noisy channel without degradation ◊ Easy to manipulate ◊ Less expensive communication ◊ More reliable ◊ Flexible ◊ Compatibility with other digital systems ◊ Integrated networks	◊ Sampling Error ◊ Digital signals require greater bandwidth than analogue to transmit the same information.
Analogue	◊ Uses less bandwidth ◊ More accurate	◊ The effects of random noise can make signal loss and distortion impossible to recover.

Outline

- Introduction to Digitization
- **Digital Controllers**
- Implementing Control Modes
- PID Tuning
- Summary

Digital Controllers

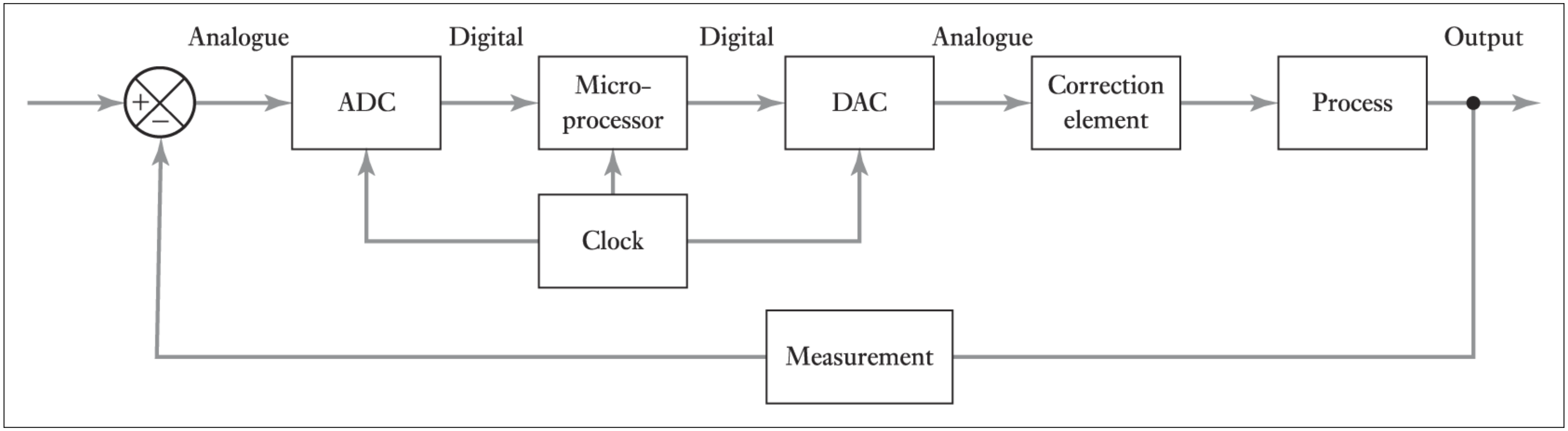


Digital closed-loop control system

- The controller (microprocessor) receives inputs from sensors, executes control programs and provides the output to the correction elements.
- Such controllers requires inputs in digital form using an analogue-to-digital converter (ADC).
- A clock supplies a pulse at regular time intervals and dictates when samples of the controlled variable are taken by the ADC.

Digital Controllers

- **Digital Controller's Cycle of Events**

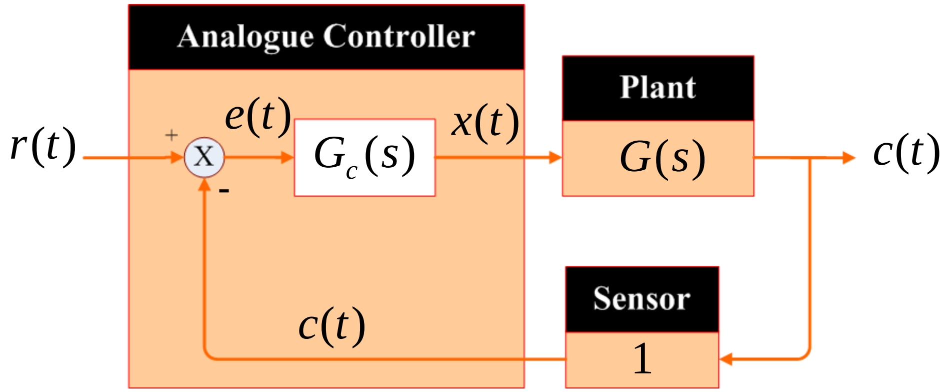


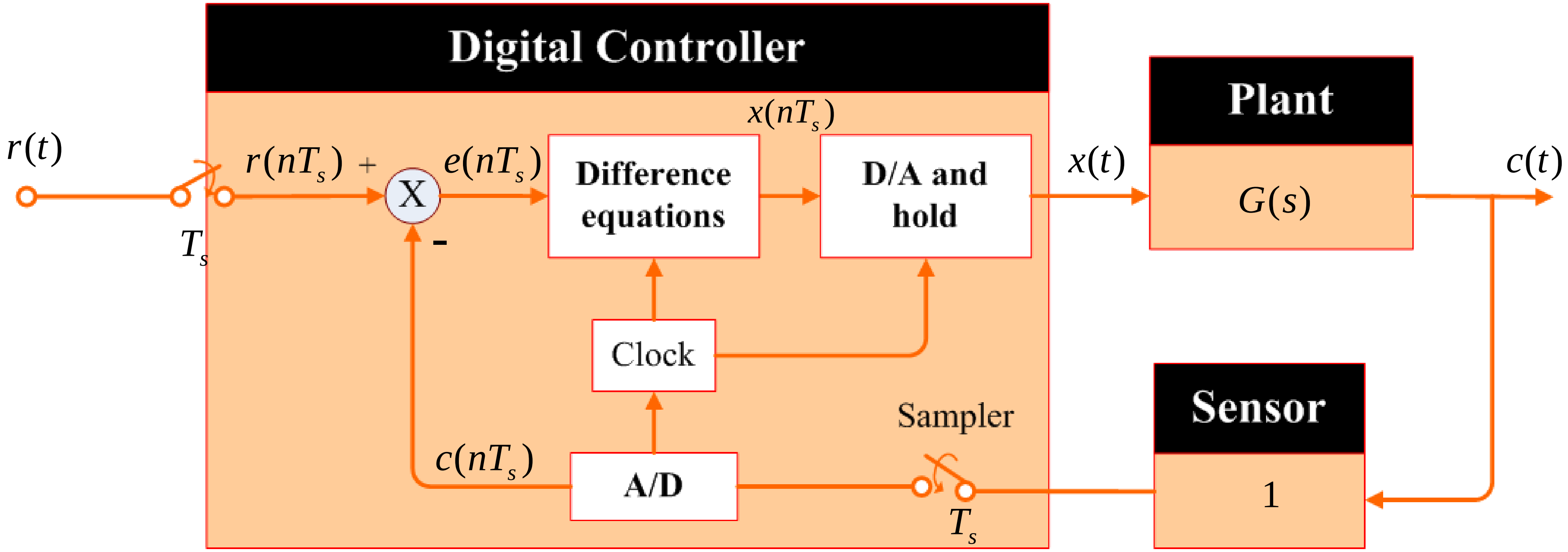
Digital closed-loop control system

1. Samples the measured value.
2. Compares it with the set value and establishes the error.
3. Carries out calculations based on the error value and stored values of previous inputs and outputs to obtain the output signal.
4. Sends the output signal to the DAC.
5. Waits until the next sample time before repeating the cycle.

Digital Controllers

- **Digital Controller versus Analogue Controllers**





Digital Controllers

- **Digital Controller versus Analogue Controllers**

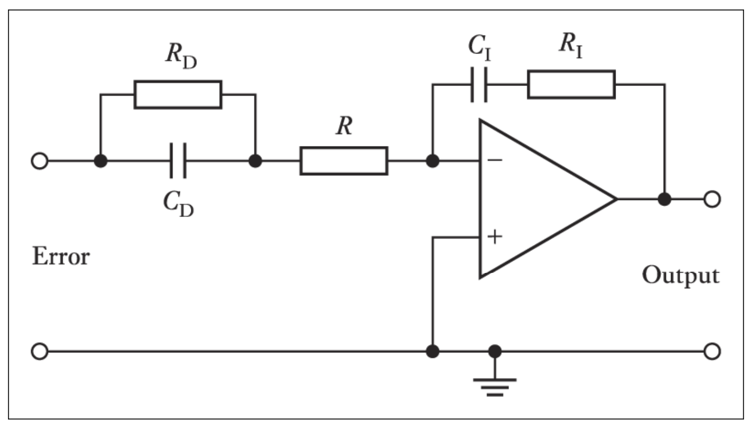
  ◇ **Form of Controlling Action**

  Digital controllers have the advantage over analogue controllers that the form of the controlling action, e.g. proportional or three-mode, can be altered by purely a change in the computer software. **No change in hardware** or electrical wiring is required. Indeed the control strategy can be altered by the computer program during the control action in response to the developing situation.

PID Algorithm

```
IF Mode = 'AUTO' THEN
Err=SetP- Input
IF Action = 'DIRECT' THEN
Err=0 – Err
ENDIF
OutP=OutP+Gain*(Err-ErrLast+Reset*Err+Rate*(Err-ErrLast*2+ErrLastLast))
ErrLastLast=ErrLast
ErrLast=Err
ELSE
InputLast=Input
 ErrLastLast=Err
ErrLast=Err
ENDIF
IF OutP > 100 THEN OutP=100
IF OutP < 0 THEN OutP=0
```

Analogue PID



$$K_P = \frac{R_I}{R + R_D}$$

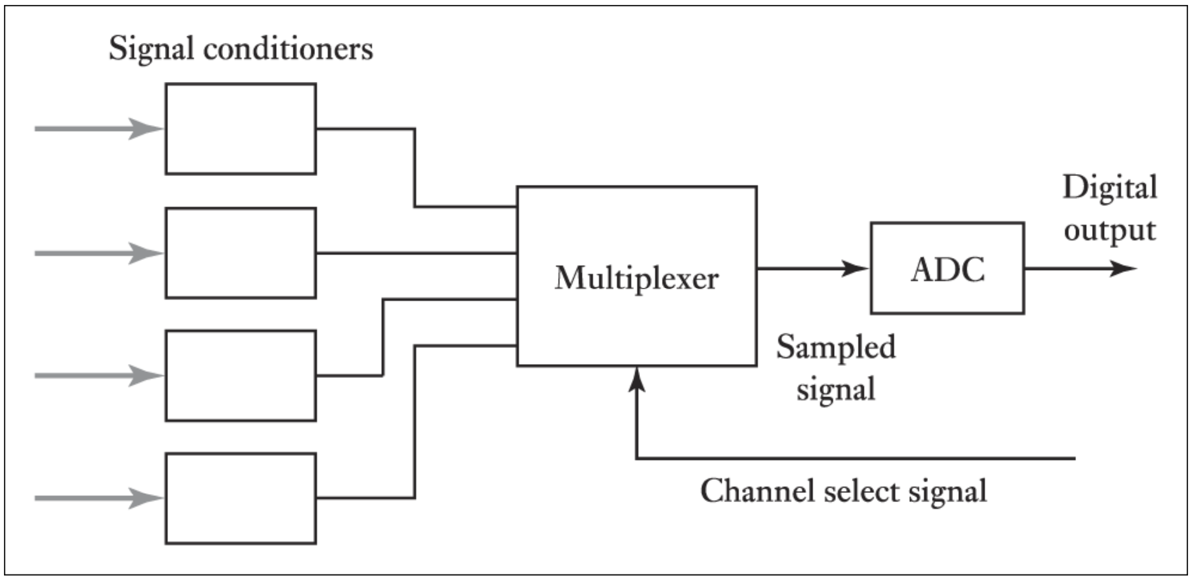
$$K_D = R_D C_D$$

$$K_I = \frac{1}{R_I C_I}$$

Digital Controllers

- **Digital Controller versus Analogue Controllers**
  - **# of controllers per process**

    With analogue control, **separate** controllers are required for each process being controlled. With a microprocessor many separate processes can be controlled by sampling processes with a **multiplexer**.



Multiplexer

Digital Controllers

- **Digital Controller versus Analogue Controllers**
  - ◇ **Accuracy**

    Digital control gives **better accuracy** than analogue control because the amplifiers and other components used with analogue systems **change their characteristics with time and temperature** and so show drift, while digital control, because it operates on signals in only the on/off mode, does not suffer from drift in the same way.

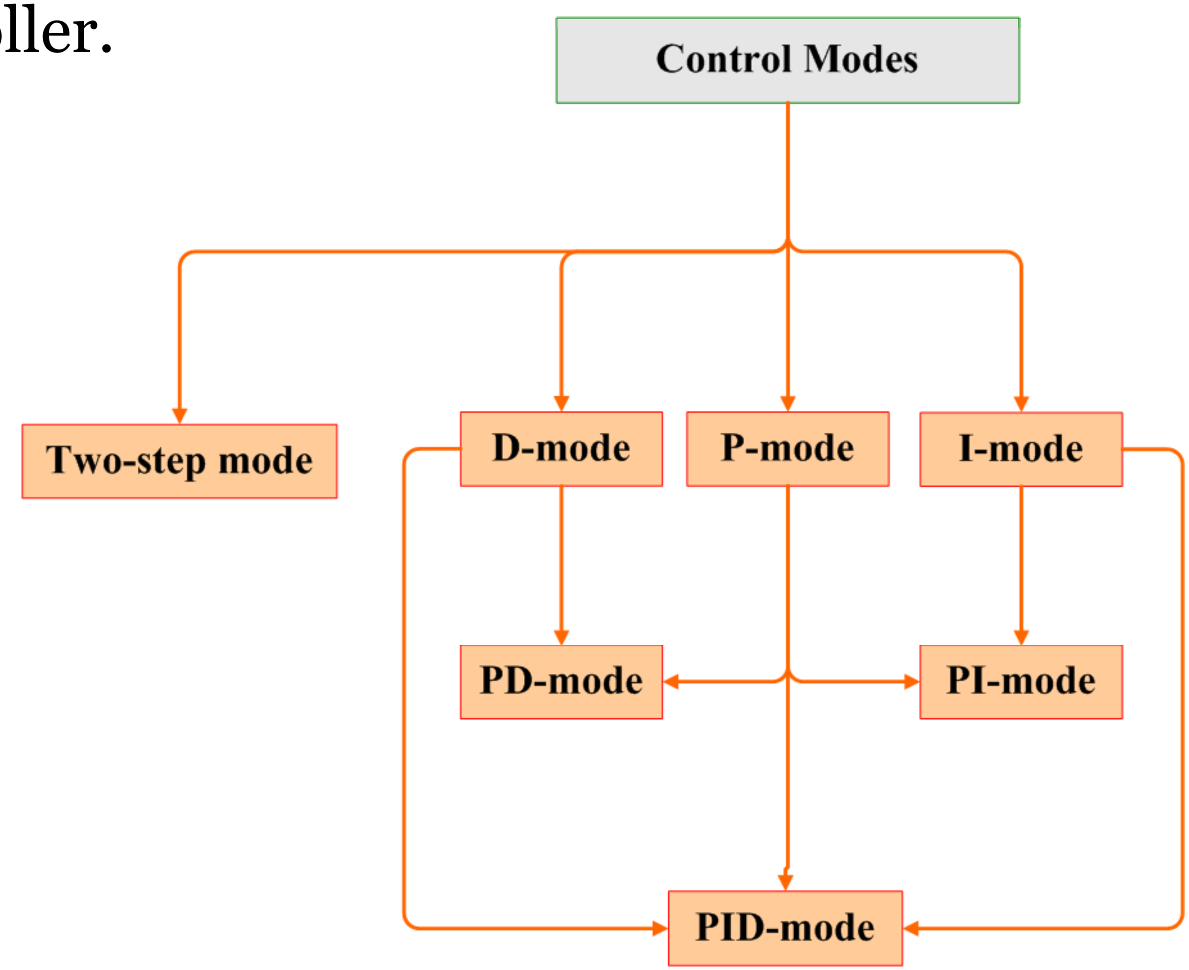
Outline

- Introduction to Digitization
- Digital Controllers
- **<u>Implementing Control Modes</u>**
- PID Tuning
- Summary

Implementing Control Modes

- In order to produce a digital controller which will give a particular mode of control it is necessary to produce a suitable program for the controller.

- The program has to indicate how the digital error signal at a particular instant is to be processed in order to arrive the required output for the following correction element.

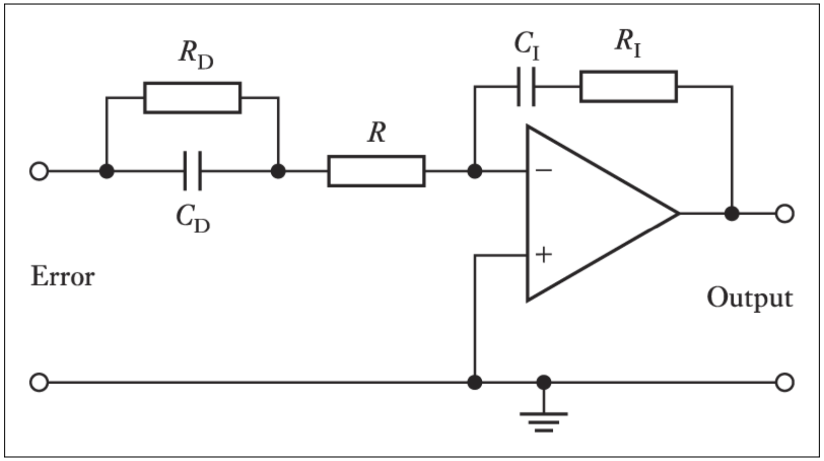


Implementing Control Modes

- For a PID analogue controller, the transfer function is

$$\text{transfer function} = K_P + sK_D + \frac{1}{s}K_I$$

$$K_P = \frac{R_I}{R + R_D},\ K_D = R_D C_D,\ K_I = \frac{1}{R_I C_I}$$



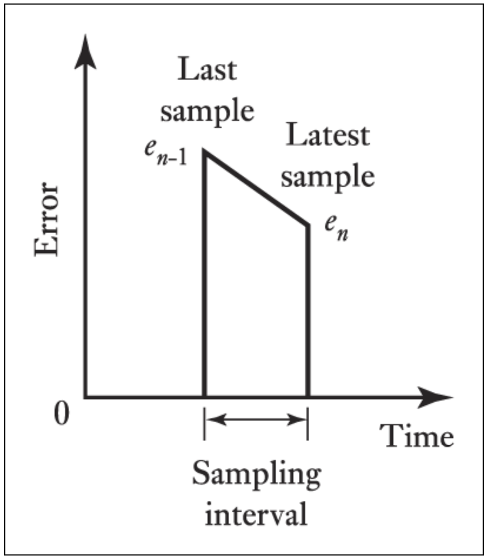
Analogue PID

◇Multiplication by s $\xrightarrow{\text{equivalent to}}$ differentiation

However, we can consider the **gradient of the time response** for the error signal at the present instant of time as being: $\frac{e_n - e_{n-1}}{T_s}$

where

e_n is the latest sample of error; e_{n-1} is the last sample of the error and T_s is the sampling interval



Error signal

Implementing Control Modes

- For a PID analogue controller, the transfer function is

$$\text{transfer function} = K_P + sK_D + \frac{1}{s}K_I$$

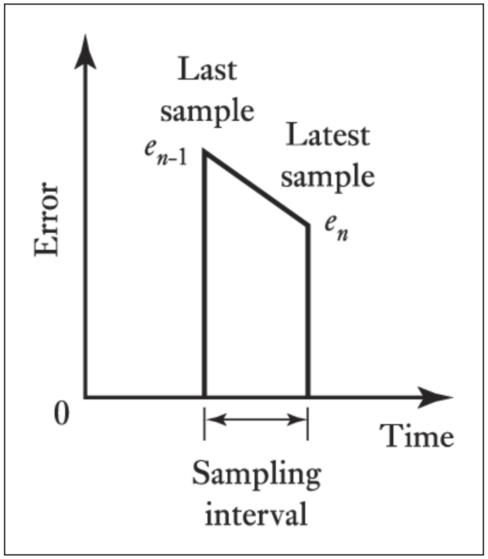
◇ Division by $s \xrightarrow{\text{equivalent to}}$ integration

However, we can consider the integral of the error at the end of a sampling period as being area under the error-time graph during the last sampling interval plus the sum of the areas under the graph for all previous sample (Int_{prev}).

If the sampling period is short relative to the times involved then the area during the last sampling interval is approximately:

$\frac{1}{2}(e_n + e_{n-1})T_s$ and the total area is:

$$\frac{1}{2}(e_n + e_{n-1})T_s + \text{Int}_{\text{prev}}$$



Error signal

Implementing Control Modes

- Now we can write **the controller output x_n** at a particular instant n given **latest error e_n** and **last error e_{n-1}** as:

$$x_n = K_P e_n + K_I \left(\frac{(e_n + e_{n-1}) T_s}{2} + \text{Int}_{\text{prev}} \right) + K_D \frac{e_n - e_{n-1}}{T_s}$$

Rearranging this equation gives:

$$x_n = A e_n + B e_{n-1} + C(\text{Int}_{\text{prev}})$$

where

$$A = K_P + 0.5 K_I T_s + K_D / T_s$$

$$B = 0.5 K_I T_s - K_D / T_s$$

$$C = K_I$$

Implementing Control Modes

$$x_n = Ae_n + Be_{n-1} + C(\text{Int}_{\text{prev}})$$

- The program for PID control thus becomes:

1. Set the values of K_P, K_I and K_D.
2. Set the initial values of e_{n-1}, Int_{prev} and the sample time T_s.
3. Reset the sample interval timer.
4. Input the error e_n.
5. Calculate x_n using the above equation.
6. Update, ready for the next calculation, the value of the previous area to $\text{Int}_{\text{prev}}+0.5(e_n+e_{n-1})T_s$.
7. Update, ready for the next calculation, the value of the error by setting e_{n-1} to e_n.
8. Wait for the sampling interval to elapse.
9. Go to step 3 and repeat the loop.

Outline

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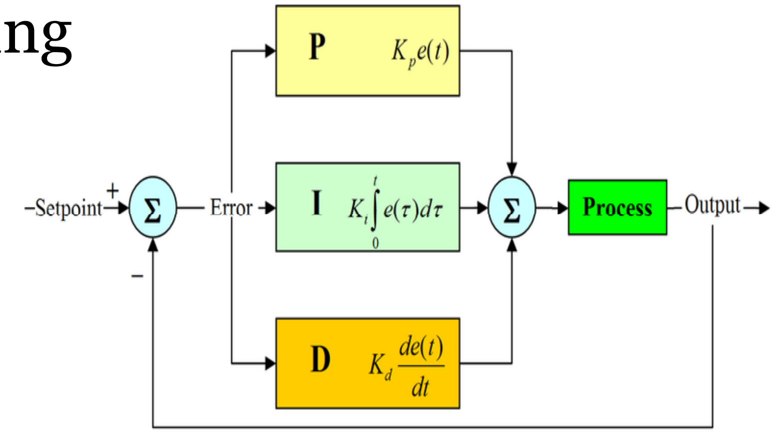
- Implementing Control Modes
- **PID Tuning**
- Summary

PID Tuning

- **Controller Tuning**
  - Tuning is the process of selecting the best controller settings.
  - With P controller, this means selecting the value of K_P
  - With PID controller, the three constants K_P, K_D and K_I have to be selected.



Effect of PID Controllers on Closed-Loop System

	P	I	D
Rise Time	Decreases	Decreases	Small Change
Overshoot	Increases	Increases	Decreases
Settling Time	Small change	Increases	Decreases
S.S. Error	Decreases	Eliminates	Small change

Note that these correlations may not be exactly accurate, because K_P, K_I, and K_D are dependent on each other. In fact, changing one of these variables can change the effect of the other two. For this reason, the table should only be used as a reference when you are determining the values for K_I, K_P and K_D.

PID Tuning

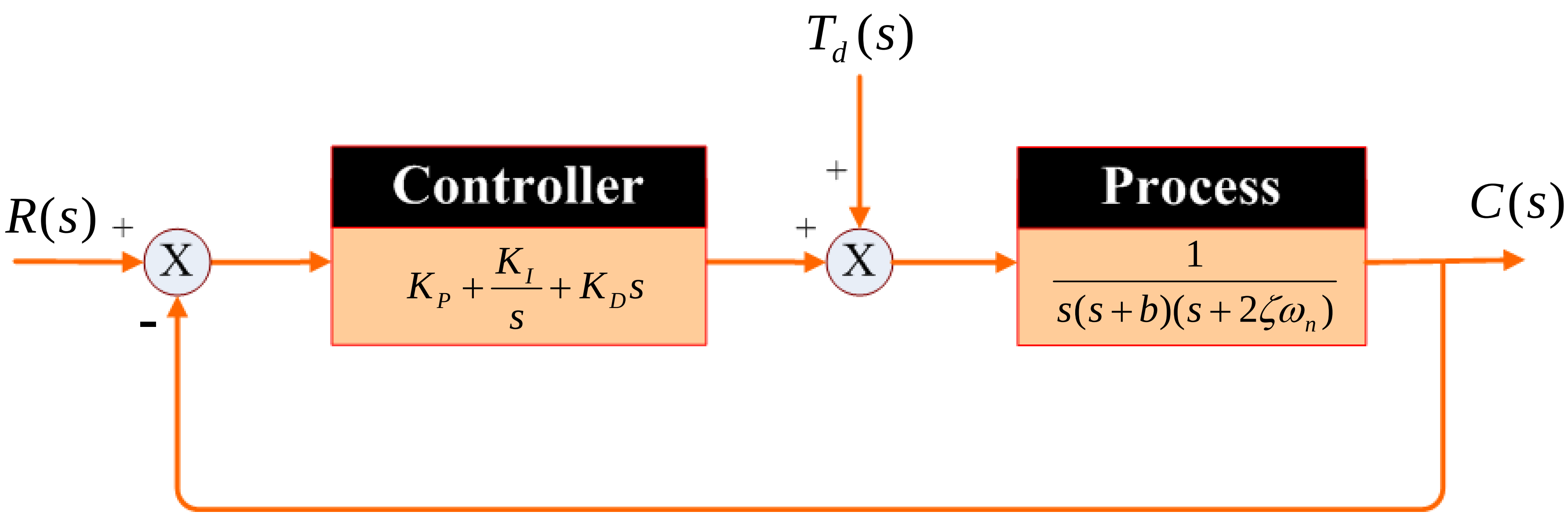
- **Manual Tuning**

1. Select a typical operating setting for the desired output, turn off integral and derivative parts, then increase K_P to maximum or until oscillation occurs.
2. If system oscillates, divide K_P by 2.
3. Increase K_D and observe behavior when increasing/decreasing the desired output by about 5%. Choose a value of K_D which gives a damped response.
4. Slowly increase K_I until oscillation starts. Then divide K_I by 2 or 3.
5. Check whether overall controller performance is satisfactorily under typical system conditions.

PID Tuning

- **Manual Tuning: Example**

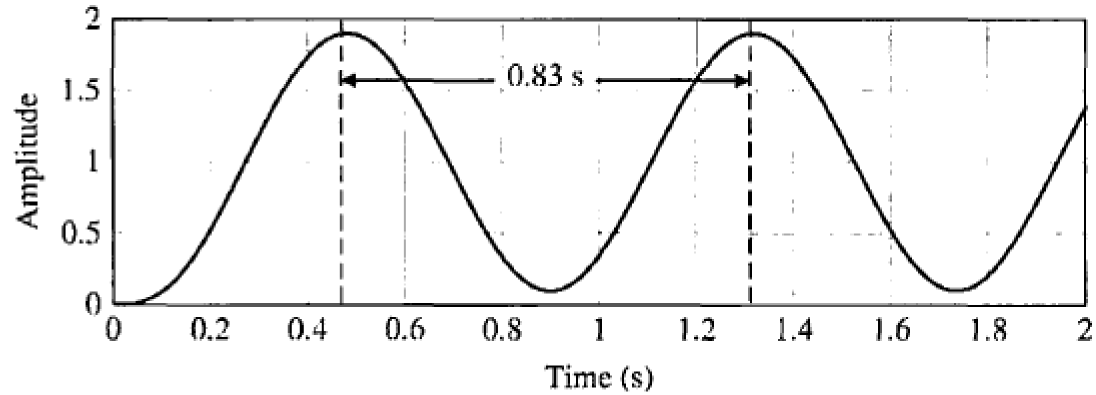
Consider the following closed-loop system



where b = 10, ζ = 0.707, and $\omega_n = 4$.

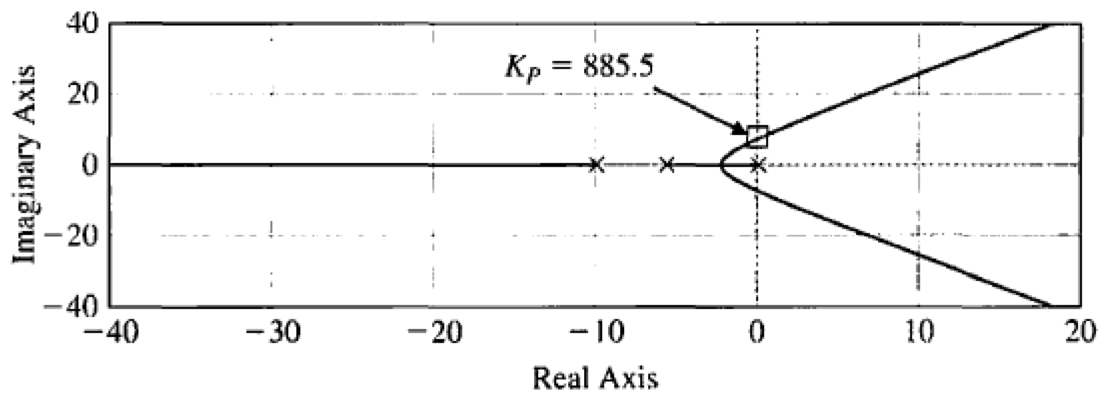
PID Tuning

- **Manual Tuning: Example (cont'd)**
  - ◇ To begin the manual tuning process, set $K_I = 0$ and $K_D = 0$ and increase K_P until the closed-loop system has sustained oscillations.
  - ◇ As can be seen in the figure, when $K_P = 885.5$, we have a sustained oscillation of magnitude A = 1.9 and period $T_c = 0.83$ s.



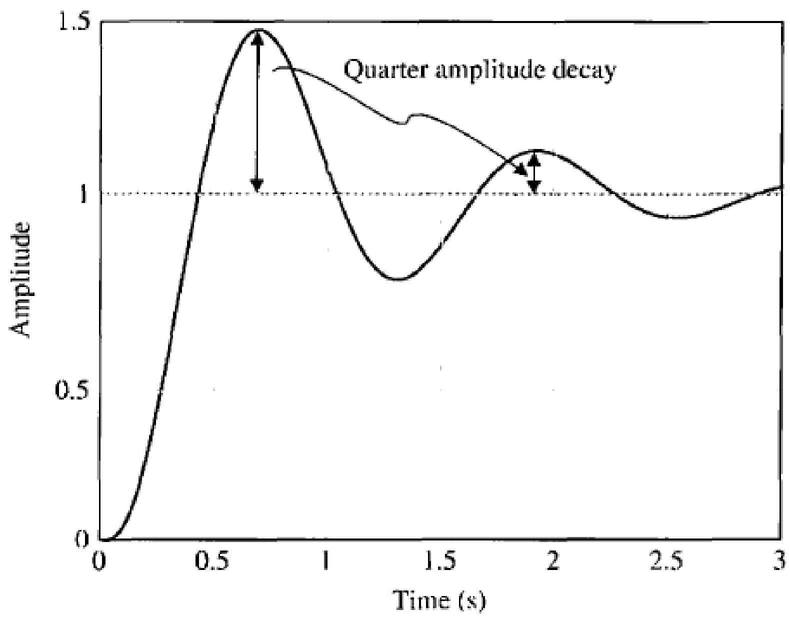
  - ◇ This root locus corresponds to the characteristic equation

$$1 + K_P\left[\frac{1}{s(s+10)(s+5.66)}\right] = 0$$



PID Tuning

- **Manual Tuning: Example (cont'd)**
  - ◇ Reduce $K_P = 885.5$ by half as a first step to achieving a step response with approximately a **quarter amplitude decay**.
  - ◇ **Quarter amplitude decay** is when the amplitude of the closed-loop response is reduced approximately to **one-fourth of the maximum value** in **one oscillatory period**.



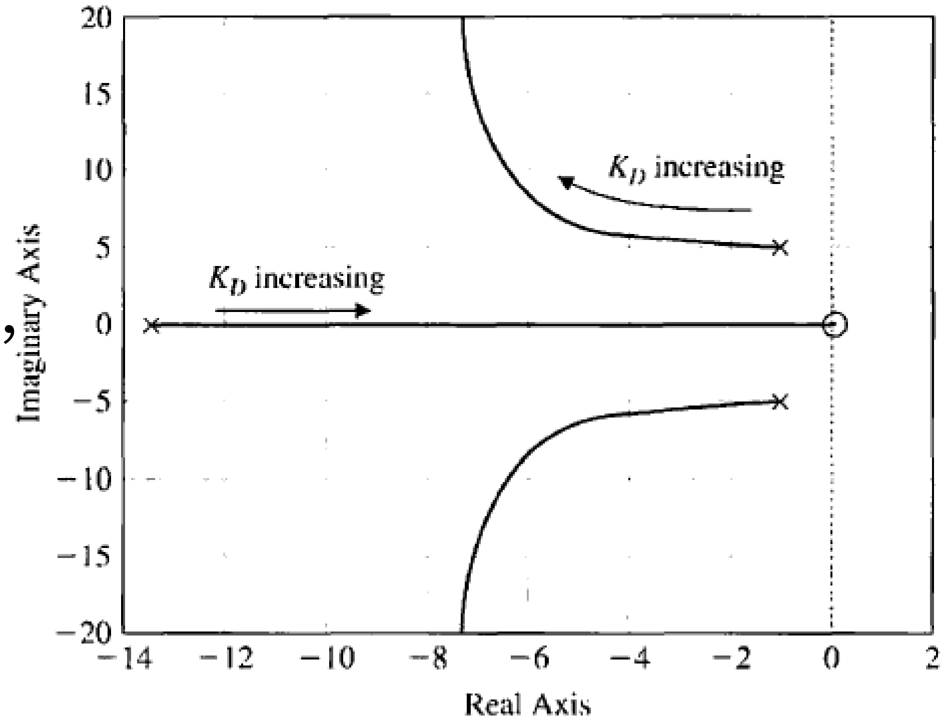
Step response with $K_P = 370$ showing the quarter amplitude decay.

We note that the peak amplitude is reduced to one-fourth of the maximum value in one period, as desired.

PID Tuning

- **Manual Tuning: Example (cont'd)**
  - ◇ The root locus for $K_P = 370$, $K_I = 0$, and $0 \le K_D < \infty$ is shown in the following figure. In this case, the characteristic equation is

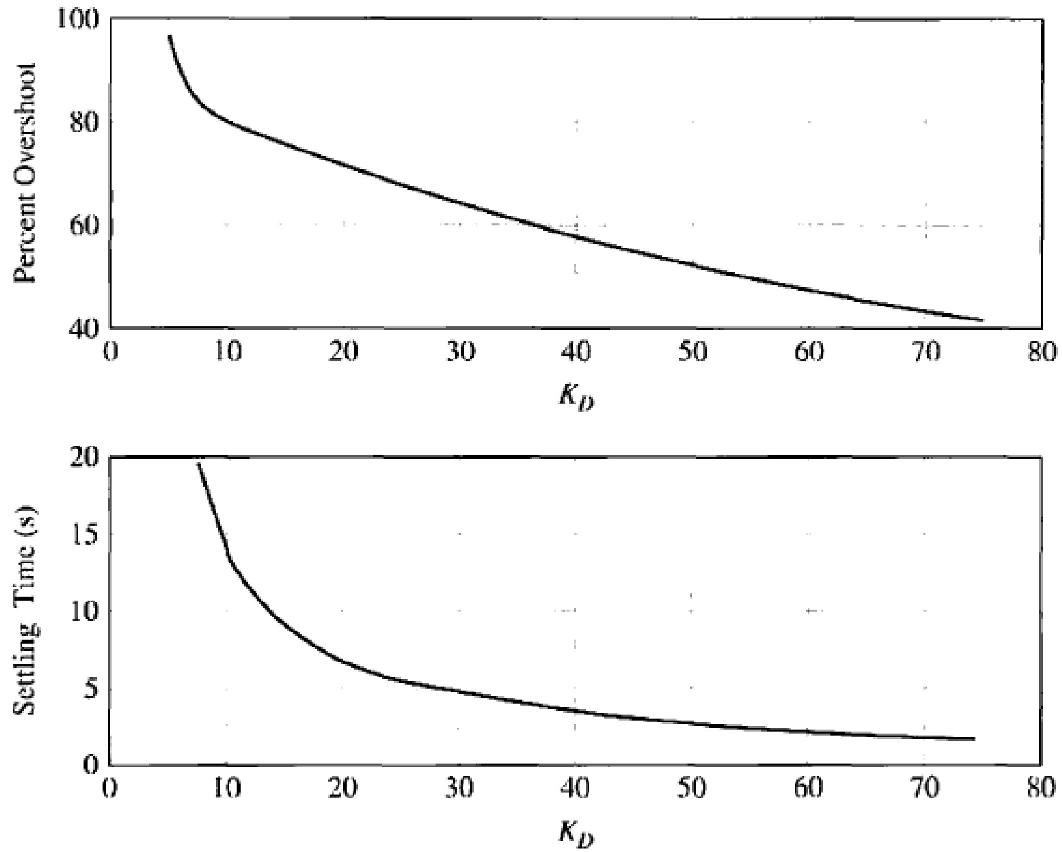
$$1 + K_D\left[\frac{s}{s(s+10)(s+5.66)+K_P}\right] = 0$$



  - ◇ As K_D increases, the closed-loop complex poles move **left**, and in doing so, increases the associated damping ratio and thereby **decreases the percent overshoot**.
  - ◇ The movement of the complex poles to the left also increases the associated $\zeta \omega_n$, thereby **reducing the settling time.**

PID Tuning

- **Manual Tuning: Example (cont'd)**
  - ◇ As K_D increases (when $K_D > 75$), the **real root** begins to **dominant** the response and become less accurate.

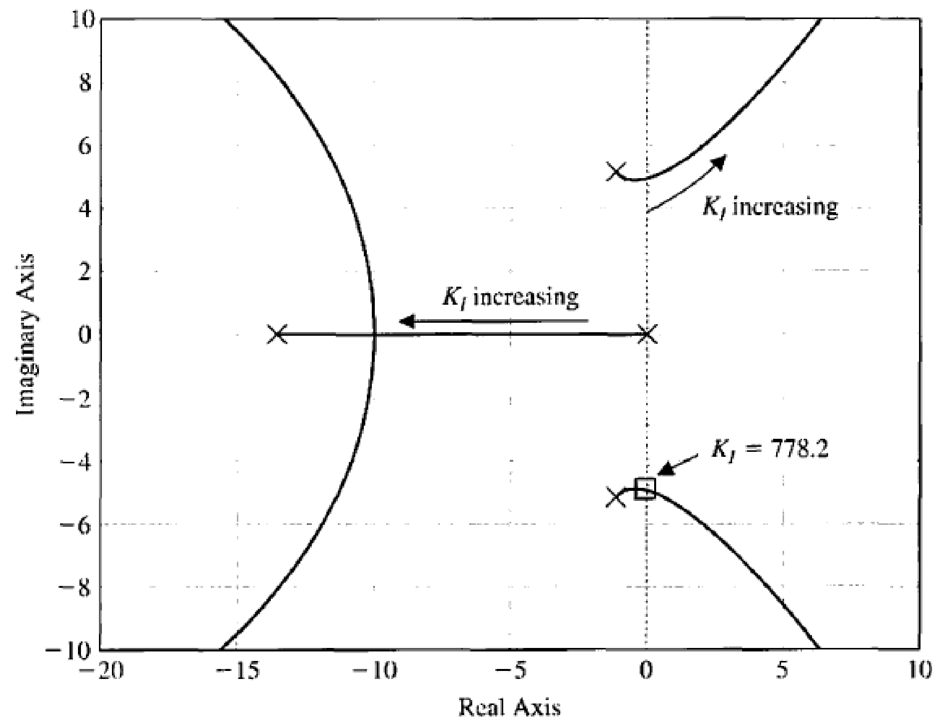


Percent overshoot and settling time with $K_P = 370$, $K_I = 0$, and $5 < K_D < 75$.

PID Tuning

- **Manual Tuning: Example (cont'd)**
  - ◇ The root locus for $K_P = 370$, $K_D = 0$, and $0 \le K_I < \infty$ is shown in the following figure. The characteristic equation is

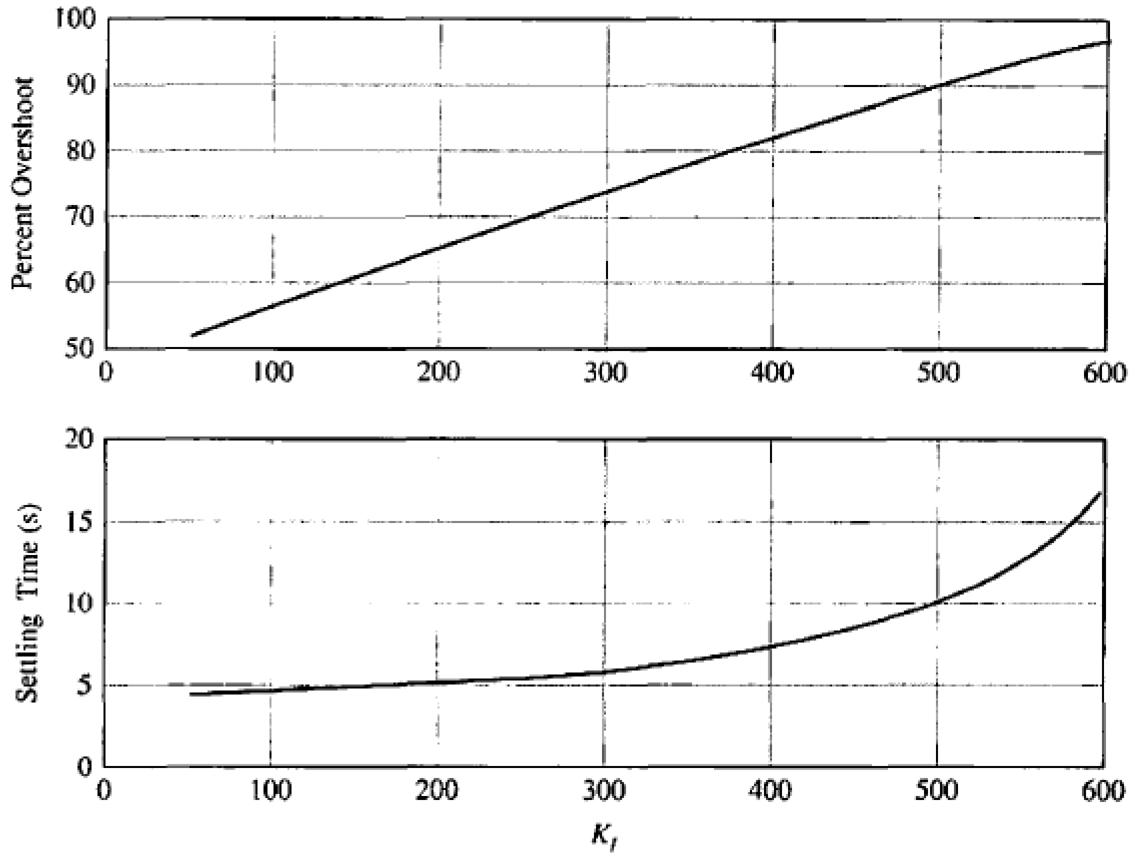
$$1 + K_I\left[\frac{1}{s((s+10)(s+5.66)+K_P)}\right] = 0$$



  - ◇ As K_I increases, the closed-loop complex pair poles **move right**. This decreases the associated damping ratio and thereby **increasing the percent overshoot**.
  - ◇ In fact, when $K_I = 778.2$, the system is marginally stable with closed-loop poles at $s = \pm 4.86j$.

PID Tuning

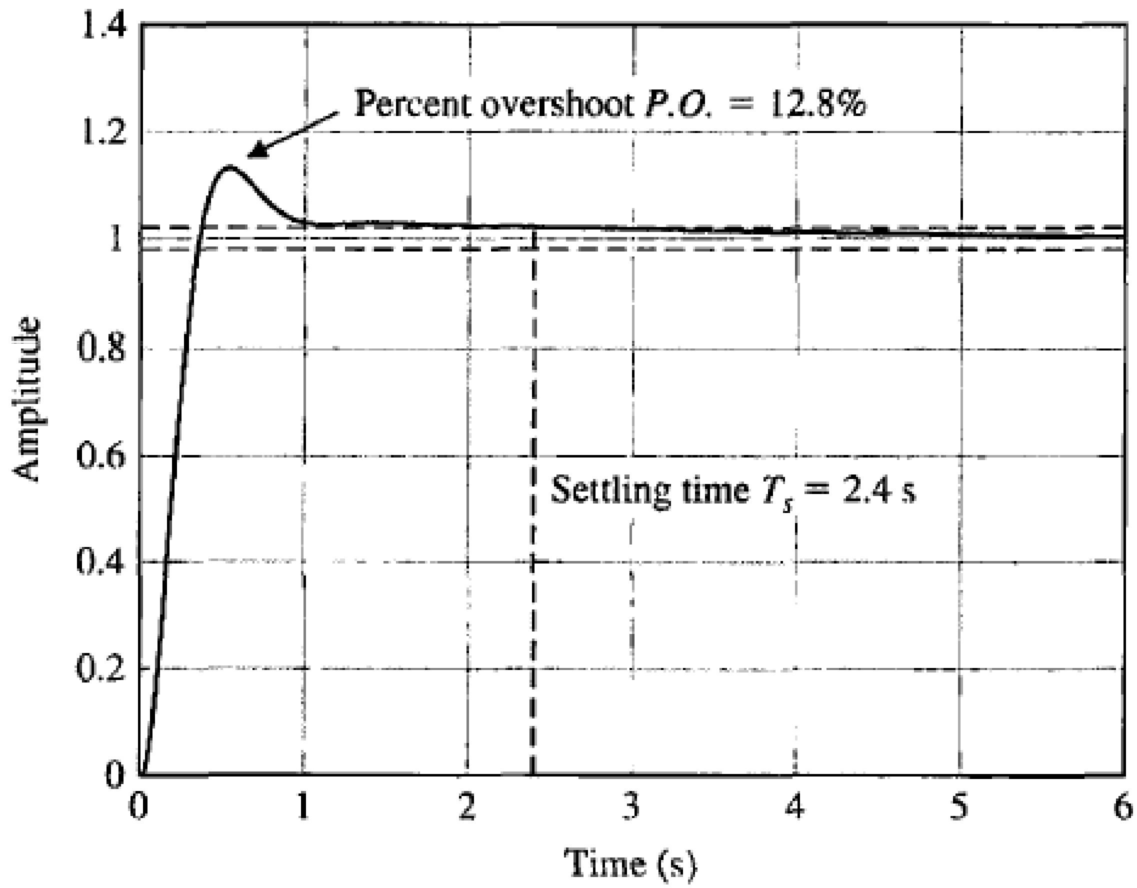
- **Manual Tuning: Example (cont'd)**
  - ◇ The movement of the complex poles to the right also decreases the associated $\zeta\,\omega_n$, thereby **increasing the settling time**.



Percent overshoot and settling time with $K_P = 370$, $K_D = 0$, and $50 \le K_I < 600$.

PID Tuning

- **Manual Tuning: Example (cont'd)**
  - ◇ To meet the percent overshoot and settling time specifications, we can select $\mathbf{K_P = 370, K_D = 60,}$ **and** $\mathbf{K_I = 100}$.



PID Tuning

- **Ziegler-Nichols PID tuning**

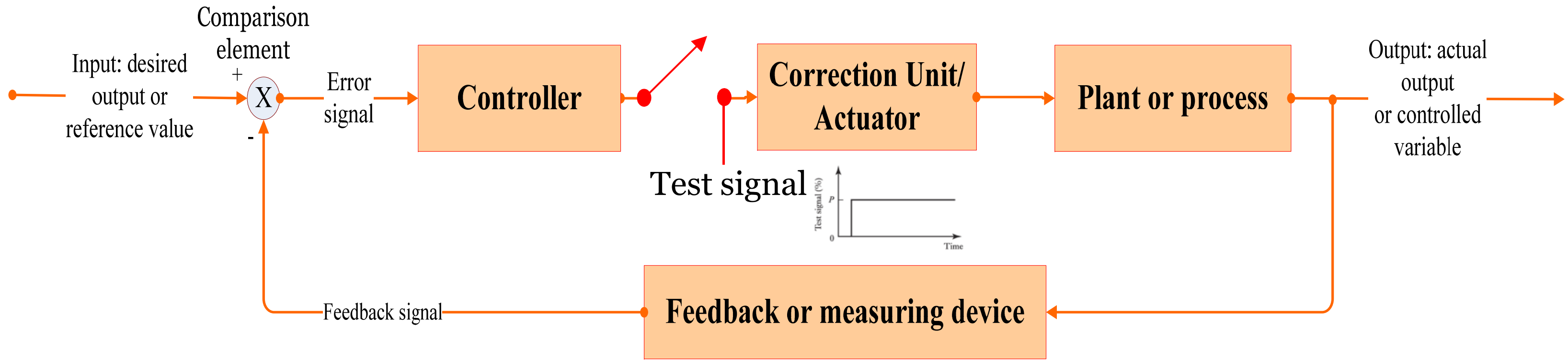
Ziegler-Nichols Tuning Methods

Open-loop Ziegler-Nichols Tuning Method
Or
Process Reaction Method

Closed-loop Ziegler-Nichols Tuning Method
Or
Ultimate Cycle Method

PID Tuning

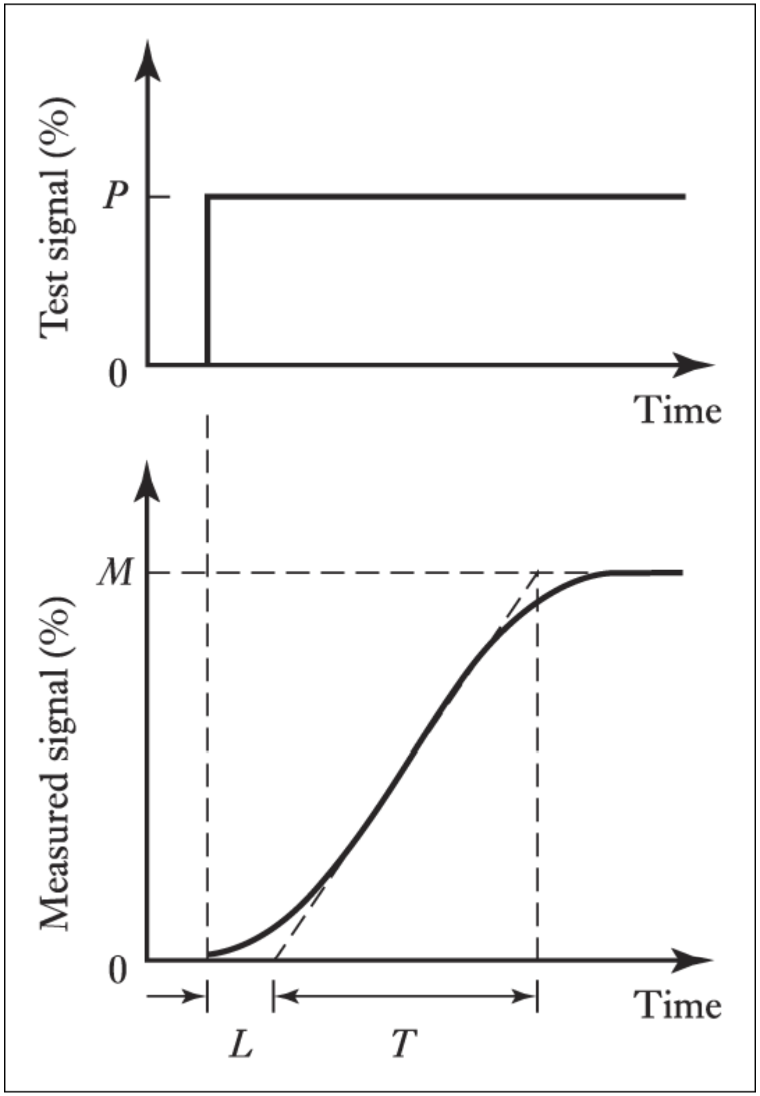
- **Ziegler-Nichols PID tuning: Process Reaction Method**
  - ◇ This approach is based on open-loop concepts relying on reaction curves.



  - ◇ The process control loop is opened (off-line), generally between the controller and the correction unit, so that no control action occurs.
  - ◇ A test input signal is then applied to the correction unit and the response of the controlled variable determined.
  - ◇ The test signal should be as small as possible.

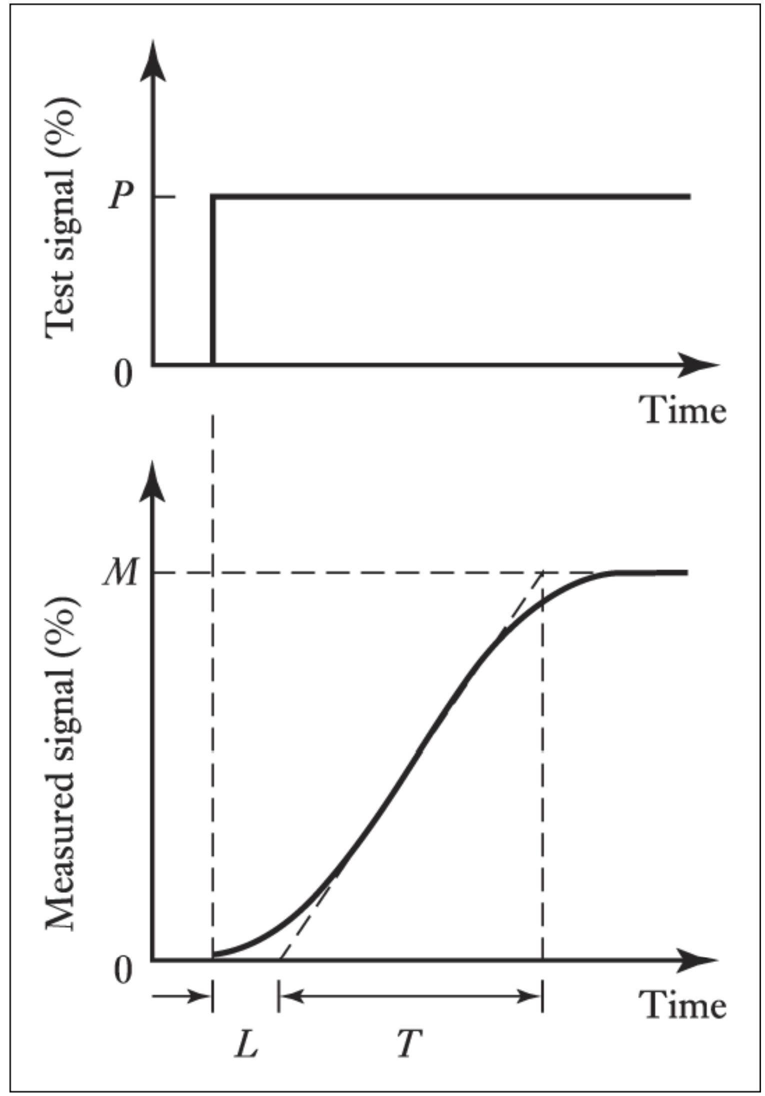
PID Tuning

- **Ziegler-Nichols PID tuning: Process Reaction Method**
  - ◇ The following figure shows the form of the test signal and a typical response (**process reaction curve**).
  - ◇ The response in the process reaction curve implies that the process is a **first-order** system with a transport delay.
  - ◇ If the actual system does **NOT** match the assumed form, then another approach to PID tuning should be considered.



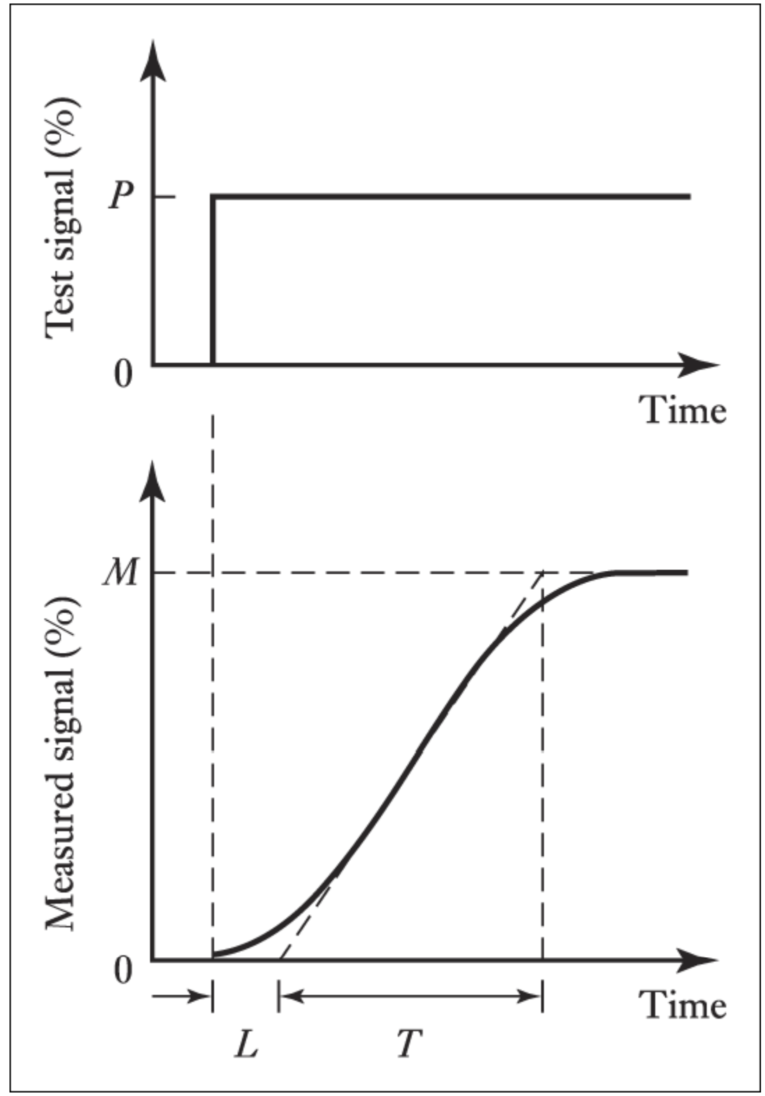
PID Tuning

- **Ziegler-Nichols PID tuning: Process Reaction Method**
  - ◇ However, if the underlying system is linear and lethargic (or sluggish and characterized by delay), the assumed model may suffice to obtain a reasonable PID gain selection using the open-loop Ziegler-Nichols tuning method.



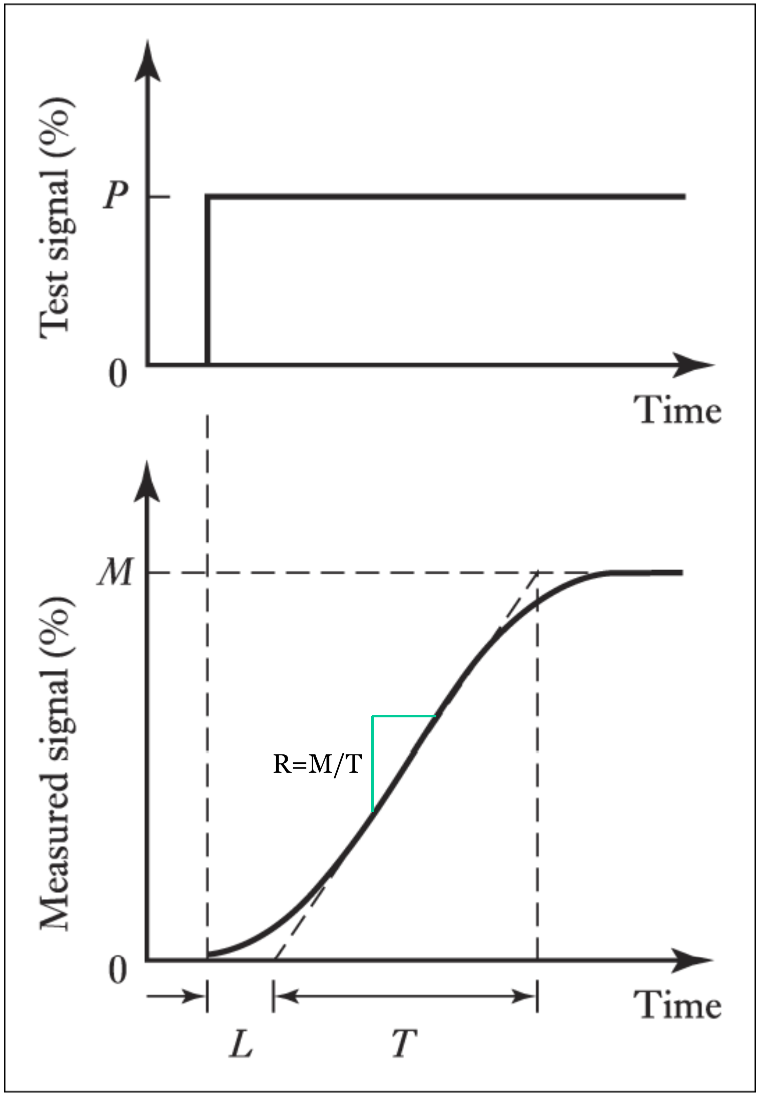
PID Tuning

- **Ziegler-Nichols PID tuning: Process Reaction Method**
  - ◇ In the **Process Reaction Curve:**
  - ◇ The test signal is a step signal with a **step size** expressed as the **percentage change P** in the correction unit.
  - ◇ The measured variable is expressed as the percentage of the full-scale range (**M** is the magnitude of the response at steady-state).



PID Tuning

- **Ziegler-Nichols PID tuning: Process Reaction Method**
  - In the **Process Reaction Curve:**
  - A tangent is drawn to give the maximum gradient of the graph R.
  - **R=M/T**
  - The time between the start of the test signal and the point at which this tangent intersects the graph time axis is termed the **Lag or Transport Delay, L**.



PID Tuning

- **Ziegler-Nichols PID tuning: Process Reaction Method**
  - ◇ **Tuning Rules:**

Process reaction curve criteria

Control mode	K_P	T_I	T_D
P	P/RL		
PI	$0.9P/RL$	$3.33L$	
PID	$1.2P/RL$	$2L$	$0.5L$

$T_I = K_P / K_I$ = integral time constant

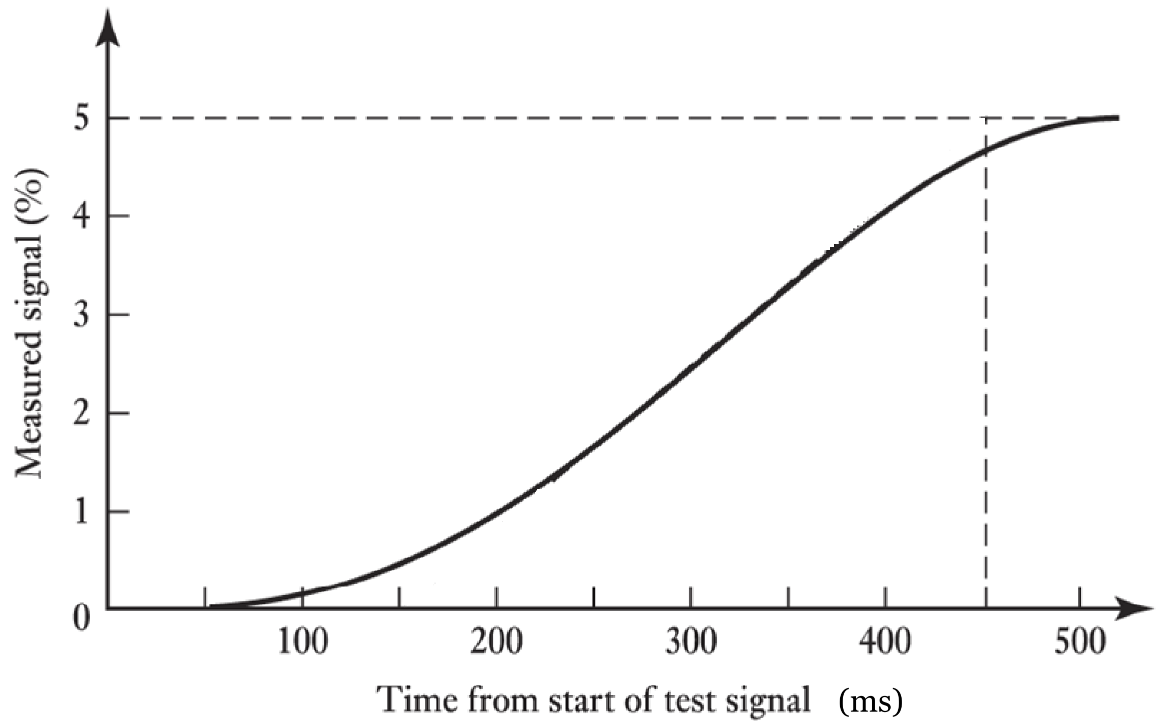
$T_D = K_D / K_P$ = derivative time constant

$R = M/T$ = maximum gradient

L = lag or transport delay

PID Tuning

- **Ziegler-Nichols PID tuning: Process Reaction Method**
  - ◇ **Example:** Determine the settings required for a three-mode controller which gave the process reaction curve shown in the following figure when the test signal was a 6% change in the control valve position.



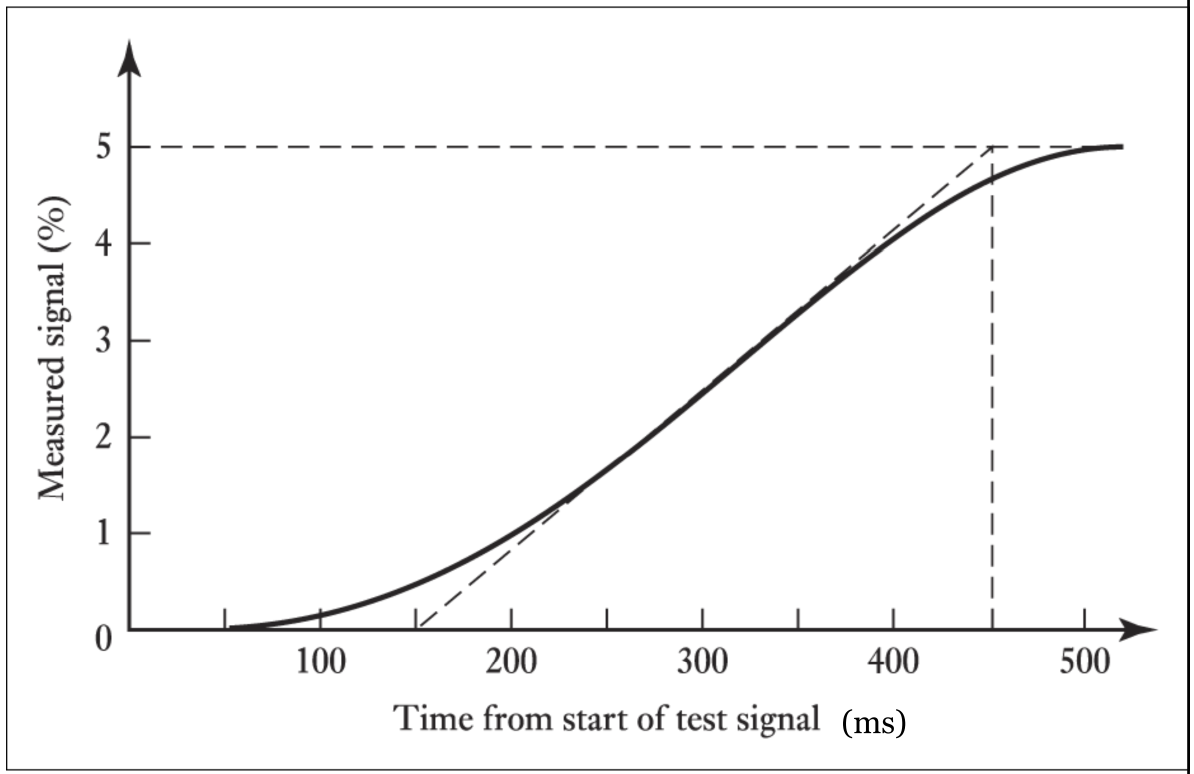
PID Tuning

- **Ziegler-Nichols PID tuning: Process Reaction Method**
  - ◇ **Solution:** Drawing a tangent to the maximum gradient part of the graph gives:
  - ◇ A lag L=150ms and a gradient R=5/300=0.017/ms
  - ◇ Then

$$K_P = \frac{1.2P}{RL} = \frac{1.2\times 6}{0.17\times 150} = 2.82$$

$$T_I = 2L = 300\text{ms}$$

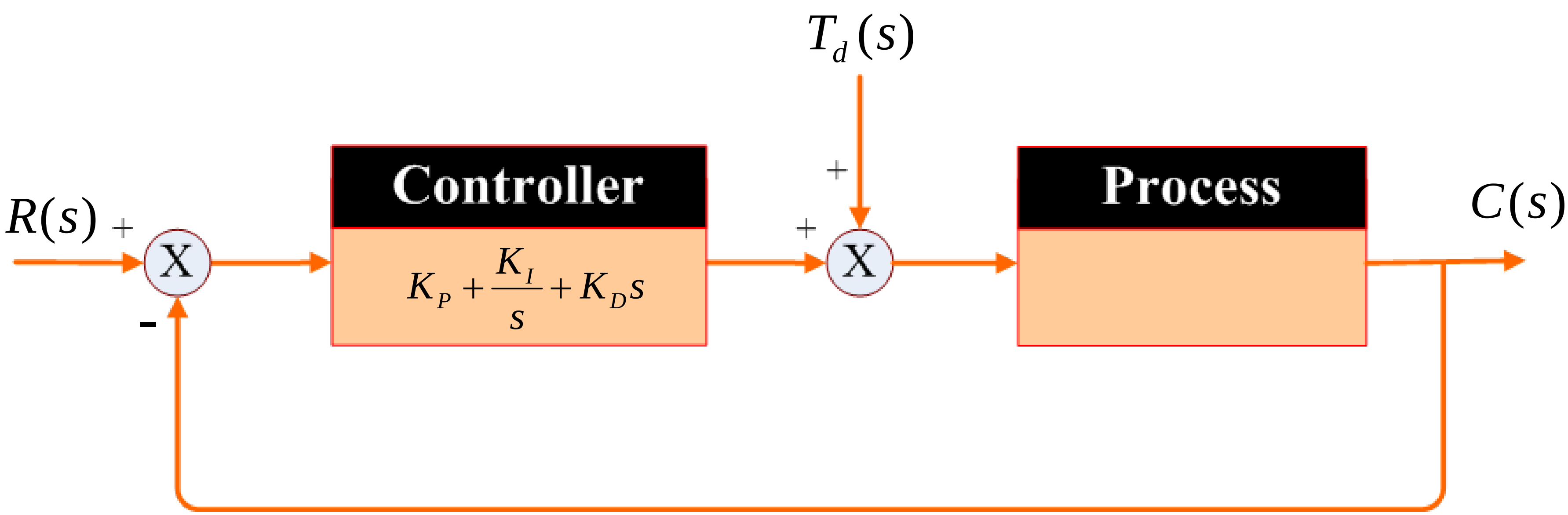
$$T_D = 0.5L = 0.5\times 150 = 75\text{ms}$$



Process reaction curve

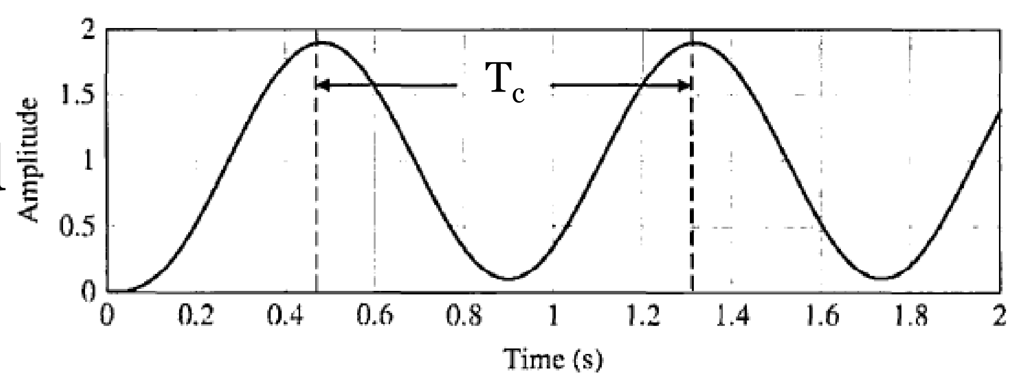
PID Tuning

- **Ziegler-Nichols PID tuning: Ultimate Cycle Method**
  - ◇ Ultimate cycle method is a closed-loop Ziegier-Nichols tuning method that considers the closed-loop system response to a step input (or step disturbance) with the PID controller in the loop.



PID Tuning

- **Ziegler-Nichols PID tuning: Ultimate Cycle Method**
  - ◊ Initially, the integral and derivative actions are first reduced to their minimum values.
  - ◊ The proportional constant K_P is set low and then gradually increased.
  - ◊ While doing this small disturbances are applied to the system.
  - ◊ This is continued until continuous oscillations occur.
  - ◊ This critical value of the proportional constant $\mathbf{K_{Pc}}$ at which this occurs is noted and the periodic time of the oscillations $\mathbf{T_c}$ measured.



PID Tuning

- **Ziegler-Nichols PID tuning: Ultimate Cycle Method**
  - ◇ Ultimate Cycle Criteria

Control mode	K_P	T_I	T_D
P	$0.5K_{Pc}$		
PI	$0.45K_{Pc}$	$T_c/1.2$	
PID	$0.6K_{Pc}$	$T_c/2.0$	$T_c/8$

PID Tuning

- **Ziegler-Nichols PID tuning: Ultimate Cycle Method**
  - ◇ **Example:** When tuning a three-mode control system by the ultimate cycle method it was found that oscillations begin when K_{Pc} is 3.33. The oscillations have a periodic time of 500 ms. What are the suitable settings for the controller?
  - ◇ **Solution:** Using ultimate cycle criteria

$$K_P = 0.6K_{Pc} = 0.6 \times 3.33 = 2.0$$

$$T_I = T_c/2.0 = 500/2 = 250\text{ms}$$

$$T_D = T_c/8 = 500/8 = 62.5\text{ms}$$

Outline

- Introduction to Digitization
- Digital Controllers
- Implementing Control Modes
- PID Tuning
- **Summary**

Summary

- A digital controller basically operates by sampling the measured value, comparing it with the set value and establishing the error, carrying out calculations based on the error value and stored values of previous inputs and outputs to obtain the output signal, outputting and then waiting for the next sample.
- In the absence of knowledge of the underlying process, a PID controller is the best controller when its parameters are properly tuned.
- Controller tuning is the process of selecting the best controller settings. It is important to point out that the use of controllers in practice does not depend on finding values of K_P, K_D, and K_I by analytical means.

Summary

- Instead, these are set by well established procedures for tuning a controller after installation. This is why experienced personnel with knowledge of the process can apply PID controllers without any formal knowledge of mathematical modeling or control theory. This important reason for the success of PID controllers should be recognized as a major advantage.
- The Ziegler-Nichols rules can be used to obtain initial controller designs followed by design iteration and refinement.
- The Ziegler-Nichols rules will not work with all plants or processes.